Exploring Mental Computation Strategies

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INTRODUCTION

The ability to perform mental computations both swiftly and flexibly remains an important mathematical skill despite the increasingly sophisticated technology that constantly surrounds us. Not only is mental computational agility considered a universally valued skill, mental computation skills provide opportunities to engage in mathematical thinking, thereby strengthening number sense and other general processes related to problem solving. Mental computation skills are a prerequisite for meaningful engagement with written algorithms. Furthermore, teaching mental computation strategies promotes creativity as learners are encouraged to think flexibly and independently.

MENTAL COMPUTATION STRATEGIES

A thorough understanding of basic number facts and arithmetical operations, as well as a solid knowledge of number bonds and number relationships, is a prerequisite to effective development of mental computation strategies. Indeed, mental computation needs to be recognised for its complexity, and as such should form an important part of the daily classroom activity right from the early years.

A number of standard strategies go by different names in different parts of the world. Nonetheless, the following discussion, which is by no means exhaustive, provides what we hope is a useful summary of some of the most commonly used mental computation strategies.

COUNTING ON & COUNTING BACK

This is a strategy which involves the simple procedure of counting on or counting back. It can be employed right from the addition and subtraction of single-digit numbers up to multiple-digit numbers. For example, in order to calculate 11 + 4 one could simply count on four from 11, i.e. 12, 13, 14, 15 (counting on in 1s). In order to calculate 19 – 3 one could count back three from 19, i.e. 18, 17, 16 (counting back in 1s). In instances where the larger number appears second, e.g. 5 + 11, learners should develop the flexibility to realise that 5 + 11 is equivalent to 11 + 5, and thus simply count on five from 11 rather than counting on eleven from 5.

The counting on and counting back strategy can also be used to count in 2s, 5s, 10s or indeed any number. For example, in order to calculate 96 – 40 learners could count back in 10s, i.e. 86, 76, 66, 56. In order to calculate 25 + 15 one could count on in 5s, i.e. 30, 35, 40. Learners need to be flexible in using this strategy depending on the nature of the computation problem presented.

PARTITIONING ONE NUMBER INTO TENS AND UNITS

For the addition and subtraction of two-digit numbers, this strategy involves keeping one number intact while partitioning the other number into tens and units. For example, to calculate 27 + 35, keep 27 intact, split 35 into 30 and 5, add 30 to 27 to get 57, and then add the remaining 5 to get 62. In order to calculate 65 – 34, keep 65 intact and split 34 into 30 and 4, subtract 30 from 65 to get 35, and then subtract the remaining 4 to get 31.

PARTITIONING INTO TENS AND UNITS

This strategy involves mentally splitting *both* numbers into tens and units before adding or subtracting them. For example, in order to calculate 37 + 24, first split the 37 into 30 and 7 and the 24 into 20 and 4. The tens are then added to give 30 + 20 = 50, and the units are added to give 7 + 4 = 11. Finally the 50 and the 11 are added to give 61. Similarly, in order to calculate 54 – 32, split 54 into 50 and 4, 32 into 30 and 2, then 50 – 30 = 20, 4 – 2 = 2, and finally 20 + 2 = 22. For larger numbers we can also split into hundreds, tens and units.

The idea can also be extended to include decimals. We often use this strategy when adding money. The money is split into Rands and cents, each part is worked out separately, and the two answers are then added together to arrive at the final answer, for example: R33.50 + R16.80 = (R33 + R16) + (R0.50 + R0.80) = R49 + R1.30 = R50.30.

ADDING AND SUBTRACTING IN STAGES

The strategy of adding or subtracting in stages is accomplished by splitting numbers into convenient components. What is 'convenient' in a given context is of course dependent on the particular numbers being added or subtracted. In general though, aiming for multiples of 10 or 100 is a useful approach. If one wanted to calculate 47 + 16 then splitting 16 into '3+13' would be sensible. So, starting with 47 we add 3 to get 50. This leaves 13 still to be added, so adding 50 and 13 gets us to the final answer of 63.

A similar approach works for subtraction. If we wanted to calculate 74 – 9 then we could mentally split 9 into '4+5'. Subtracting the 4 from 74 gives us 70, from which we can easily subtract 5 to get to the final answer of 65. If necessary, one could even increase the number of components in the split. For example, to calculate 104 – 16 we could begin by subtracting 4 from 104 to get 100, then subtract 10 from 100 to get 90, and finally subtract 2 from 90 to get to the final answer of 88. In this example the 16 has been subtracted in three stages by mentally splitting it into three components, namely '4 + 10 + 2'.

NEAR DOUBLES

This strategy can be used to compute numbers that are close to doubles or where the creation of a double can facilitate the handling of a computation problem. For sums of single-digit numbers, e.g. 5 + 6, after noticing that 6 is one more than 5 we can swiftly arrive at the answer by doubling 5 and adding 1 to get 11. For two-digit numbers, e.g. 17 + 19, we see that 19 is two more than 17 and can thus calculate the sum by doubling 17 to get 34 and then simply adding 2 to arrive at 36.

An analogous situation works for subtraction. For example, in the sum 93 – 45 one could notice that 93 is three more than double 45. Thus 93 – 45 can be thought of as '3 more than double 45' – 45, the answer to which is clearly 3 more than 45, i.e. 48. Seeing 93 as 'double 45 + 3', for example, is a useful skill to develop.

BRIDGING TO 10 AND COMPENSATING

This strategy involves bridging a number to a multiple of ten, adding/subtracting the second number to the bridged multiple of ten, and then making an adjustment to compensate for the bridging. By way of example consider the sum 28 + 35. To begin with, bridge 28 to 30. The addition of 30 to 35 can be swiftly calculated to give 65. Finally, we need to adjust the 65 down by 2 in order to compensate (i.e. cancel out) the initial bridging of 28 to 30. Subtracting 2 from 65 thus gives the final answer of 63.

A similar process works for subtraction. In order to calculate 146 – 97, first bridge 97 to 100. Subtracting 100 from 146 gives 46 which then needs to be adjusted 3 up to 49 since *too much* has been subtracted. In the sum 67 – 32, begin by bridging 32 to 30. Subtracting 30 from 67 gives 37 which then needs to be adjusted 2 down to 35 since *too little* was subtracted. A degree of critical thinking is required here to decide whether the final adjustment should be up or down.

Bridging to 10 clearly eases the calculation demands in computation problems of the type 86 – 39 since it's far easier to subtract 40 from 86 than it is to subtract 39. This strategy can also be used for calculating products. For example, to calculate 9×14 simply calculate 10×14 to get 140 and then subtract one multiple of 14 to get the final answer of 126. To calculate 48×12 simply calculate 48×10 to get 480 and then add two multiples of 48 to get 576.

CONVENIENTLY ADJUSTING BOTH NUMBERS

Consider the calculation 48 + 23. By adjusting the 48 up by 2 and the 23 down by 2 we can conveniently manipulate the calculation to the form 50 + 21. While this has the same answer as the original 48 + 23, it is easier to calculate mentally. This strategy is particularly useful when subtracting one number from another since if both numbers are adjusted up by the same amount or if both numbers are adjusted down by the same amount, the difference between them remains unchanged. By way of example, in order to calculate 96 – 38 simply adjust both numbers up by 2 to arrive at the equivalent calculation 98 – 40. Subtracting 40 from 98 to arrive at an answer of 58 is far simpler than subtracting 38 from 96 directly. Learners should be encouraged to be flexible with this strategy and seek ways to use it to maximum advantage. For example, to calculate 247 – 198 one should notice that 198 is close to a multiple of 100. Adjusting both numbers up by 2 thus yields 249 – 200 which has the same difference as the original question but which is far easier to calculate mentally.

GROUPING COMPATIBLE NUMBERS

This strategy involves looking for numbers whose sum or difference, or product or quotient, is easy to calculate mentally. Consider the sum 43 + 24 + 17. Since 43 and 17 when added result in a round multiple of 10, they can be considered as being compatible. We can thus carry out the addition process by adding 43 and 17 to get 60, and then finally adding on the 24 to get to 84. Essentially we have reorganized the original calculation into a more convenient form, i.e. (43 + 17) + 24. When calculating products, keep a lookout for pairs of numbers whose product is a multiple of 10 or 100. For example, to calculate $2 \times 34 \times 5$ we can simplify the calculation significantly with the following re-grouping: $34 \times (2 \times 5) = 34 \times 10 = 340$.

MULTIPLICATION IN STAGES

A complicated product can often be simplified by carrying out the process in stages. This is a particularly useful strategy when the stages are simply repeated doublings. For example, consider the product 15×8. We can think of the 8 as $2 \times 2 \times 2$ and thus proceed by doubling 15 to get 30, doubling 30 to get 60, and finally doubling 60 to get the final answer of 120. Division by 8, for example, could also be carried out by a process of successive halvings.

HALVING AND DOUBLING

This is a strategy used in multiplication problems where a computation problem is transformed into a less demanding one by halving one number whilst simultaneously doubling the other. Consider the following multiplication: 16×3. By halving 16 to 8 and simultaneously doubling 3 to 6 we can transform 16×3 into 8×6 which is far easier to calculate. The process of halving and doubling can also be carried out more than once depending on the nature of the problem. For example: $28 \times 25 = 14 \times 50 = 7 \times 100 = 700$.

DOUBLING

This is a strategy which can transform seemingly difficult division problems into manageable computation problems. For example, in order to calculate $33 \div 1\frac{1}{2}$, we can simply double both the dividend as well as the divisor to arrive at the equivalent division problem $66 \div 3$ which can be calculated far more easily. As with the strategy 'halving and doubling', this strategy can also be carried out repeatedly.

MENTAL IMAGE OF PEN-AND-PAPER ALGORITHM

This strategy involves carrying out a mental computation problem by mimicking a pen-and-paper method in one's mind through a mental image – e.g. adding two 2-digit numbers by mentally visualizing them one below the other, adding the units, performing a carry-over if necessary, and finally adding the digits in the tens column.

CONCLUDING COMMENTS

In addition to becoming proficient in the use of specific mental computation strategies, it is important that learners also become flexible in using these strategies. The richer the mental toolbox of strategies available, the more flexible learners should become in tackling computational problems. For example, a problem like 8×45 can be handled in any number of different ways through mental computation. One could for example think of the 8 as $2\times2\times2$ and hence simply double 45 to get 90, double it again to get 180, and double it a third time to get 360. Alternatively one could split the 45 into 40 and 5 and then proceed by multiplying 40 by 8 to get 320, 5 by 8 to get 40, and then adding the 320 and 40 to get 360. A third method could be to bridge 45 to 50, multiply 50 by 8 to get 400, and then compensate for the bridging by subtracting 8 multiples of 5 (i.e. 40) from 400 to get 360. A different approach could be to bridge 8 to 10, multiply 45 by 10 to get 450, and then compensate by subtracting two multiples of 45 (i.e. 90) from 450 to get 360.

The strategies discussed in this article by no means represent an exhaustive list, and the names of the strategies are certainly not definitive since many of them go by different names. Nonetheless, exposing learners to these strategies and encouraging them to use them on a regular basis, both consciously and flexibly, is likely to strengthen their number sense as well as general processes related to problem solving. In addition, learners should be given opportunities to develop their own strategies. As a classroom activity, give learners a set of mental calculations and ask them to record the short-cuts they used for each calculation. Collate these and generate a classroom discussion around the different strategies used. A great deal of value can be gained from learners sharing and discussing their individual strategies through verbalizing their thought processes and explaining their solution methods.